Redistribution and Fiscal Uncertainty Shocks

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Abstract

This paper studies the macroeconomic impact of an uncertainty shock on fiscal policy in a dynamic general equilibrium framework. Motivated by the observation that many fiscal policies are redistributive and that a sizable fraction of U.S. households do not own capital, I introduce household heterogeneity in the form of limited capital market participation. I show that household heterogeneity significantly magnifies the aggregate effect and induces co-movement of macroeconomic variables in a contraction that is generated by a fiscal uncertainty shock. This is because the limited capital market participation model captures individual uncertainty about redistribution that is absent in representative agent models. When agents are ambiguity averse, this uncertainty about redistribution has first-order effects because it shows up as heterogeneous worst-case scenarios. As a result, the model matches the empirical responses of macro variables to fiscal uncertainty shocks better than the representative agent counterpart.

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1 Introduction

A popular view among policymakers and business economists is that changes in uncertainty about fiscal policy are important factors in explaining macroeconomic fluctuations, including the Great Recession and the post-Great Recession period since 2009. A recent work by Baker et al. (2016) confirms this view. They construct new indexes of policy uncertainty using newspaper coverage and find that uncertainty about fiscal policy and economic policy in general have increased substantially over the post-Great Recession period. Using vector autoregressive (VAR) models, they show that innovations to policy uncertainty are associated with a sizable decline in industrial production and employment. What drives this large impact of policy uncertainty shocks on economic activity? In this paper, I propose a transmission mechanism that magnifies the recessionary effect of an increase in fiscal uncertainty and demonstrate that it can account for some salient empirical features of the responses of macroeconomic variables to fiscal uncertainty shocks.

This paper is motivated by the observation that many fiscal policies are redistributive. I call a fiscal policy redistributive when there are winners and losers in a sense that the policy makes some agents strictly worse off while other agents are strictly better off. For example, raising capital income tax might harm those who have capital income but households who do not hold capital may be better off because they may receive more transfers due to the increased government revenue. Similarly, a reduction in consumption tax may not actually be beneficial to households who do not hold capital if it leads to lower transfers but households with capital, who consume more than households without capital, might be better off. To investigate the aggregate implications of this redistribuational nature of fiscal policies, I introduce household heterogeneities in a parsimonious manner by considering limited capital market participation in the spirit of Galí et al. (2007). In contrast to standard models where the representative household holds capital stocks, I assume that a subset of population does not participate in the capital market. This is a natural assumption, since we know from studies such as Mankiw and Zeldes (1991) and Vissing-Jørgensen (2002) that only a subset of U.S. households hold stocks and their behavior is considerably different from that of households who do not hold stocks. I call those who participate in the capital market “capital holders” and those who do not “non-capital holders”. The standard representative agent assumption can be obtained as a special case when the share of non-capital holders is zero.

I assume that agents have recursive multiple prior preferences and perceive uncertainty not only as risk but also as ambiguity (Knightian uncertainty). Under this preference representation, agents lack confidence in assigning probabilities to relevant events and act as if they evaluate plans according to the worst-case scenario drawn from a set of multiple beliefs. An increase in the set of beliefs implies a loss of confidence. In this paper, I focus on confidence about future path of fiscal instruments: government spending, capital and labor income taxes, and consumption taxes. As in Bianchi et al. (2017) and Ilut and Saijo (2018), the belief sets are tied to the measured volatilities of these fiscal instruments. The multiple prior utility was axiomatized by Gilboa and Schmeidler (1989). Epstein and Schneider (2003) introduce a recursive version that is consistent with dynamic optimization.
instruments and are parameterized by intervals of conditional means. In turn, as in Fernández-Villaverde et al. (2015) and Born and Pfeifer (2014), each fiscal instrument is allowed to display time-varying volatility in its innovations. Hence, a fiscal volatility shock means an increase in the width of the intervals and thus implies first-order effects of changes in uncertainty.

I embed these two features — household heterogeneity in the form of limited capital market participation and (Knightian) uncertainty shocks about fiscal policy — into the standard New Keynesian business cycle framework of Christiano et al. (2005) to study its quantitative implications. Government spending, capital and labor income taxes, and consumption tax follow exogenous AR(1) processes and government bonds and lump-sum transfers adjust to satisfy the government budget constraint. I then use impulse-response-matching methods to estimate the structural parameters from the VAR impulse responses to fiscal uncertainty shocks. In the data, a two-standard-deviation increase in the volatility to innovations to capital income tax generates a 0.4 percent reduction in output and similar drops in consumption, investment, and hours. In addition real wages and nominal interest rates decline and the economy experiences a mild deflation. The limited capital market participation model matches the data well. As in the data, the model reproduces sizable and simultaneous fall in output, consumption, investment, and hours while also successfully matching the dynamics of prices such as real wages, inflation, and the federal interest rate. In contrast, the representative-agent counterpart of the model in which all agents participate in the capital market misses key features of the data. In particular, macro quantities such as output and hours fall too little, consumption increases rather than decreases, and real wages stay constant.

To understand the mechanism, first consider the representative agent model. An increase in uncertainty about the capital tax rate increases the width of the set of the conditional mean for the one-period-ahead capital tax and thus the representative household acts as if future capital tax rates are higher. Due to lower after-tax return on capital, households reduce their investment. In contrast, consumption increases because it becomes cheaper relative to investment. Low investment reduces the household’s incentive to supply labor. The increase in consumption, however, counteracts this by mitigating the decline in overall aggregate demand and thus equilibrium employment and output decline mildly.

Next, consider a model with limited capital market participation. The key feature of the limited capital market participation model is that the worst-case beliefs are heterogeneous. Capital holders fear a higher future capital tax rate. Non-capital holders, in contrast, act as if the future capital tax is lower. Lower capital tax makes non-capital holders worse off since, assuming transfers are equally distributed across households, lump-sum transfers are lower (lump-sum taxes are higher) because of lower government revenue. As a result of this perceived negative income effect, non-capital holders reduce their consumption. Because the consumption cut by non-capital holders outweighs the mild consumption increase by capital holders, aggregate consumption declines. Thus, aggregate demand declines and hence markups increase due to sticky prices. The increase in markups leads to lower wages and labor and as a result output falls significantly. To sum up, in the heterogeneous household...
model, non-capital holders perceive a negative income effect because the model captures individual uncertainty about redistribution that is absent in the representative agent model. Because this individual uncertainty manifests itself as heterogeneous worst-case scenarios, household heterogeneity has first-order effects: in contrast to the representative agent model, a capital income tax ambiguity shock generates a sizable fall in output, investment, consumption, and hours.

I provide several additional results. First, using my framework I can easily study the effects of one-sided fiscal uncertainty shocks. For example, a lack of clarity about the extent of a tax cut might trigger an increase in uncertainty about a tax reduction but not about a tax increase. I find that an increase in up-side and down-side uncertainty both generate contractions but the output effect of the increase in up-side uncertainty is more persistent and is as five times as large as the increase in down-side uncertainty. Second, I show that the main finding that limited capital market participation amplifies the impact of capital income tax uncertainty shocks on economic activity is robust under a plausible range of the capital market participation rate. Finally, I show that when the economy is stuck at the zero lower bound (ZLB) on the nominal interest rate the macroeconomic effects of a capital income tax uncertainty shock are substantially amplified. This is because under the ZLB the central bank cannot lower the interest rate to counteract low aggregate demand due to the consumption cut by non-capital holders.

The household heterogeneity is also critical to understanding uncertainty shocks to other fiscal instruments. Whether the limited capital market participation significantly alters the transmission of a fiscal uncertainty shock or not depends on whether that fiscal instrument is redistributive or not. For example, in the case of a government spending uncertainty shock, the worst-case scenario of both capital holders and non-capital holders is higher government spending. This is because government spending is financed by a lump-sum tax that is imposed equally on both types of agents. Hence the impact of a government spending uncertainty shock is very similar under both full and limited capital market participation settings. The same goes for a labor income tax uncertainty shock; since an increase in the labor income tax generates a recession that outweighs the redistribution among agents, higher labor tax makes both types of households worse off. Hence, the effect of a labor income tax uncertainty shock is not significantly affected by household heterogeneity. In contrast, consider a consumption tax uncertainty shock. In the representative agent model, an increase in uncertainty generates negligible effects on economic activity. In the limited capital market participation model, the worst-case scenarios are heterogeneous: capital holders, who consume more than non-capital holders, fear high consumption tax while non-capital holders fear low consumption tax because they would be worse off due to lower transfers. Thus, as in the capital tax uncertainty shocks case, non-capital holders perceive a negative income effect, which leads to lower aggregate demand. As a consequence, the model predicts that, as in the VAR responses, a consumption tax uncertainty shock generates a sizable adverse effect on aggregate quantities. I conclude that household heterogeneity due to limited capital market participation is an important propagation mechanism for fiscal uncertainty.

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2Fernández-Villaverde et al. (2015) and Born and Pfeifer (2014), in contrast, only study two-sided fiscal uncertainty shocks.
shocks that is consistent with key features of the data.

1.1 Relation to the literature

This paper is related to three strands of literature. First, this paper is most closely related to Fernández-Villaverde et al. (2015) and Born and Pfeifer (2014). Both papers study the effects of fiscal uncertainty shocks using a representative-agent New Keynesian model. I introduce limited capital market participation and show that household heterogeneity has important implications for aggregate outcomes. Bachmann et al. (2015) and Bretscher et al. (2017) also study the interaction of household heterogeneity and fiscal uncertainty. Their focuses are different from those in this paper. In Bachmann et al. (2015), they use an incomplete-market real business cycle model a l’a Krusell and Smith (1998) to study the welfare and distributional consequences of permanently eliminating fiscal uncertainty. Bretscher et al. (2017) estimate a New Keynesian model with a savers-spenders type heterogeneity to analyze the implications of fiscal uncertainty shocks on the term structure of interest rates and bond risk premia. My goal is to investigate the short-run impact of fiscal uncertainty shocks on macroeconomic variables. More broadly, this paper is part of a rapidly-growing literature that is interested in how other types of uncertainty shocks affect macroeconomic fluctuations. Examples in this literature include Bloom (2009), Fernández-Villaverde et al. (2011), Basu and Bundick (2017), Arellano et al. (2018), Bachmann and Bayer (2013), and Christiano et al. (2014).

I also relate to the emerging literature on Knightian uncertainty and business cycles. For example, Ilut and Schneider (2014) show that Knightian uncertainty shocks to aggregate TFP can explain a substantial fraction of aggregate fluctuations. For other applications, see Bianchi et al. (2017) and Ilut and Saijo (2018). Two papers are particularly relevant in this literature. Methodologically, I build on the work by Ilut et al. (2016), who develop an algorithm to solve linear, dynamic, heterogeneous agent models with Knightian uncertainty and study the properties of a borrower-lender model. I apply their method to a New Keynesian environment with heterogeneous households who are ambiguity averse. Michelacci and Paciello (2017) consider the impact of imperfectly credible monetary policy announcements in a New Keynesian model where, as in Ilut et al. (2016), households are ambiguity averse and have heterogeneous net financial wealth. In their paper, the announcement of a future policy loosening could lead to a contraction due to the perceived negative income effect because the economy behaves as if the aggregate net worth is lower. In this paper, limited capital market participation amplifies the impact of capital income tax uncertainty shocks because the economy behaves as if the government budget constraint does not hold in the aggregate, since both types of agents act as if they will be losers.

Finally, in this paper I introduce household heterogeneity to a New Keynesian environment by considering limited asset market participation in the tradition of Galí et al. (2007), Bilbie (2008), Broer et al. (2016), Saijo (2016), Walsh (2016) and others use the framework to study the transmission mechanism of various aggregate shocks. This paper characterizes the impact of fiscal uncertainty shocks in a setting with limited capital market participation.
2 Model

To evaluate the impact of fiscal uncertainty shocks, I study a New Keynesian business cycle model in the tradition of Christiano et al. (2005) and Smets and Wouters (2007). The framework is a natural environment for my quantitative analysis since it has now become the foundation of applied research in both academic and government institutions. I introduce two additional features. First, households are ambiguity averse as in Ilut and Schneider (2014) and face Knightian uncertainty about fiscal policies. Second, I assume limited capital market participation in the spirit of Galí et al. (2007). This allows me to consider meaningful heterogeneity on the household side in a parsimonious way. In the following, letters without a time subscript refer to steady-state values and letters with hats to log-deviations from the steady states.

2.1 Fiscal shocks and fiscal uncertainty shocks

I first specify the true data generating process for fiscal rules that describe the law of motion of four main fiscal policy instruments: the ratio of government spending to output, $g_t$, consumption tax rate, $\tau_{c,t}$, labor income tax rate, $\tau_{h,t}$, and capital income tax rate, $\tau_{k,t}$. Each instrument follows the process described below:

$$\hat{x}_t = (1 - \rho_x)\bar{x} + \rho_x\hat{x}_{t-1} + \phi_{x,Y}\hat{Y}_{t-1} + \phi_{x,B}\ln\left(\frac{B^g_{t-1}}{Y_{t-1}}/\frac{B^g}{Y}\right) + \exp(\sigma_{x,t})u_{x,t}, \quad u_{x,t} \sim N(0, 1), \quad (1)$$

for $x \in \{g, \tau_{c}, \tau_{h}, \tau_{k}\}$. The fiscal rule (1) embeds two feedbacks. First, there is an “automatic stabilizer” component that allows an instrument to respond to the log-deviation of lagged output, $Y_{t-1}$, from the steady state ($\phi_{g,Y} < 0$ and $\phi_{\tau_{x,Y}} > 0$). Second, an instrument may also respond to the log-deviation of debt-to-GDP ratio, $B^g_{t-1}/Y_{t-1}$, from its steady-state value ($\phi_{g,B} < 0$ and $\phi_{\tau_{x,B}} > 0$). Following Fernández-Villaverde et al. (2015) and Born and Pfeifer (2014), I allow for stochastic volatility in innovations of each fiscal instrument. I assume the stochastic volatility component follows an exogenous AR(1) process:

$$\sigma_{x,t} = (1 - \rho_{\sigma_x})\bar{\sigma}_x + \rho_{\sigma_x}\sigma_{x,t-1} + \zeta_x u_{x,t}, \quad (2)$$

where $u_{x,t} \sim N(0, 1)$.

Agents are not confident about the innovations to the fiscal instrument processes (1). As a result, they entertain a set of conditional means $\mu_{x,t}$ centered around the benchmark mean of zero:

$$\hat{x}_{t+1} = (1 - \rho_x)\bar{x} + \rho_x\hat{x}_t + \phi_{x,Y}\hat{Y}_t + \phi_{x,B}\ln\left(\frac{B^g_t}{Y_t}/\frac{B^g}{Y}\right) + \mu_{x,t} + \exp(\sigma_{x,t})u_{x,t+1}, \quad (3)$$

where $\mu_{x,t}$ is a set of intervals $\mu_{x,t} \in [-a_{x,t}, a_{x,t}]$. Agents only consider the conditional means $\mu_{x,t}$ that
are sufficiently close to the long run average of zero in the sense of relative entropy:

\[
\frac{\mu_{x,t}^2}{2\{\exp(\sigma_{x,t})\}^2} \leq \frac{1}{2}\eta^2,
\]

where the left hand side is the relative entropy between two normal distributions that share the same variance \(\{\exp(\sigma_{x,t})\}^2\) but have different means (\(\mu_{x,t}\) and zero), and \(\eta \geq 0\) is a parameter that controls the size of the entropy constraint. The entropy constraint (4) results in a set \([-a_{x,t}, a_{x,t}]\) for \(\mu_{x,t}\) in (3) that is given by

\[
a_{x,t} = \eta \exp(\sigma_{x,t}).
\]

Hence, when volatility increases, agents become less confident and consider a wider set of means. The advantage of linking ambiguity to volatility as in (5) is that it allows me to impose discipline on the magnitude and variations on the ambiguity process since time-varying volatility of fiscal instruments can be estimated from data. This is particularly important for applying time-varying ambiguity to fiscal policies since survey data on expectations about future fiscal instruments that could possibly be used to measure ambiguity is quite limited.\(^3\)

### 2.2 Households

There is a unit mass of ambiguity-averse households. A fraction \(1 - \chi\) of households have access to capital markets. I call them capital holders (hence the superscript “c”). The rest of the \(\chi\) fraction of households do not hold capital. I call them non-capital holders (hence the superscript “n”). The standard representative agent model obtains when \(\chi = 0\).

Capital holders have recursive multiple priors utility (Epstein and Schneider 2003). The multiple priors specification captures agents’ lack of confidence in probability assessments. Collect the exogenous state variables in a vector \(s_t \in S\). A household consumption plan \(C^c\) gives, for every history \(s^t\), the consumption of the final good \(C^c_t(s^t)\) and the amount of hours worked \(H^c_t(s^t)\). For a given consumption plan \(C^c\), utility is defined recursively by

\[
U^c_t(C^c; s^t) = \ln(C^c_t - bC^c_{t-1}) - \frac{(H^c_t)^{1+\phi}}{1 + \phi} + \beta d_t \min_{\mu_{x,t} \in [-a_{x,t}, a_{x,t}], \forall x} E^\mu[U^c_{t+1}(C^c; s^t, s_{t+1})],
\]

where \(b\) is the consumption habit, \(\phi\) is the inverse of Frisch labor supply elasticity and \(\beta\) is the steady-state subjective discount factor. \(d_t\) is a discount factor shock that follows

\[
\ln d_t = \rho_d \ln d_{t-1} + \epsilon_{d,t}, \quad \epsilon_{d,t} \sim N(0, \sigma_d^2).
\]

The discount factor shock is used in the zero lower bound exercise in the quantitative analysis. For

\(^3\)For example, the Survey of Professional Forecasters only asks for forecasts of future government spending. Baker et al. (2016) provide policy uncertainty indexes for government spending and taxes, but do not provide those for separate tax categories.
each fiscal instrument $x$, agents entertain a set of conditional means $\mu_{x,t} \in [-a_{x,t}, a_{x,t}]$. Because agents are not willing to integrate over their beliefs and narrow down each set to a singleton, they take a precautionary approach and act as if the true data generating process is drawn from the worst-case belief that minimizes their expected continuation utility. The model reduces to the standard rational expectations model when each set is a singleton.

Capital holders maximize their utility subject to the budget constraint:

$$
(1 + \tau_{c,t})P_tC_t^c + P_tI_t^c + P_tB_t^c \leq (1 - \tau_{h,t})P_tW_{i,t}H_{i,t}^c + (1 - \tau_{k,t})P_tR_t^k K_{t-1}^c
$$

$$
+ P_t\tau_{k,t} \delta K_{t-1}^c + R_{t-1}P_{t-1}B_{t-1}^c + Q_{i,t}^c + D_t^c + P_tT_t^c,
$$

and the capital accumulation equation:

$$
K_t^c = (1 - \delta)K_{t-1}^c + \left\{1 - \frac{\kappa}{2} \left( \frac{I_t^c}{I_{t-1}^c} - \gamma \right)^2 \right\} I_t^c,
$$

where $P_t$ is the price level, $I_t^c$ is investment, $B_t^c$ is the real bond holding, $W_t$ is the real wage, $R_t^k$ is the real rental rate of capital, $R_t$ is the nominal interest rate, $D_t^c$ are dividends from intermediate firms, and $T_t^c$ is a lump-sum transfer (or tax if it takes a negative value). I assume that households buy securities, whose payoff $Q_{i,t}^c$ is contingent on whether they can re-optimize their wage. The existence of state-contingent securities ensures that capital holders are homogeneous with respect to consumption and asset holdings, even though they are heterogeneous with respect to the wage rate and hours because of the idiosyncratic nature of the timing of wage re-optimization. I assume that both capital holders and non-capital holders trade the securities among themselves. This implies that consumption and asset holdings could be different between the two groups.

Non-capital holders also have recursive multiple priors utility:

$$
U_t^n(C_t^n; s^t) = \ln(C_t^n - bC_{t-1}^n) - \frac{(H_{i,t}^n)^{1+\phi}}{1 + \phi} + \beta d_t \min_{\mu_{a,t} \in [-a_{x,t}, a_{x,t}], \forall x} E^\mu[U_{t+1}^n(C_{t+1}^n; s^{t+1}, s_{t+1})],
$$

and their budget constraint is

$$
(1 + \tau_{c,t})P_tC_t^n + P_tB_t^n \leq (1 - \tau_{h,t})P_tW_{i,t}H_{i,t}^n + R_{t-1}P_{t-1}B_{t-1}^n + Q_{i,t}^n + P_tT_t^n - \frac{v}{2} \left( \frac{B_{t}^n}{Y_t} \right)^2 P_tY_t.
$$

Non-capital holders do not participate in the capital market but have access to risk-less bonds. Non-capital holders are subject to the same discount factor shocks as capital holders. The last term in the budget constraint is the quadratic bond-holding cost, whose size is controlled by the parameter $v > 0$. This cost is a technical device that induces stationarity in equilibrium bond holding (Schmitt-Grohe and Uribe 2003).
2.3 Firms

In each period $t$, the final goods, $Y_t$, are produced by a perfectly competitive representative firm that combines a continuum of intermediate goods, indexed by $j \in [0, 1]$, with technology

$$Y_t = \left[ \int_0^1 Y_{j,t} \frac{\theta_p - 1}{\theta_p} dj \right]^{\theta_p - 1}.$$ 

$Y_{j,t}$ denotes the time $t$ input of intermediate good $j$ and $\theta_p$ controls the price elasticity of demand for each intermediate good. The demand function for good $j$ is

$$Y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\theta_p} Y_t,$$

where $P_t$ and $P_{j,t}$ denote the price of the final good and intermediate good $j$, respectively. $P_t$ is related to $P_{j,t}$ via the relationship

$$P_t = \left[ \int_0^1 P_{j,t}^{1-\theta_p} dj \right]^{\frac{1}{1-\theta_p}}.$$

The intermediate-goods sector is monopolistically competitive. In period $t$, each firm $j$ rents $K_{j,t}$ units of capital stock from the household sector and buys $H_{j,t}$ units of aggregate labor input from the employment sector to produce intermediate good $j$ using technology

$$Y_{j,t} = z_t K_{j,t}^{\alpha} \left( \gamma H_{j,t} \right)^{1-\alpha},$$

where $z_t$ is an aggregate technology shock that follows

$$\ln z_t = \rho_z \ln z_{t-1} + \epsilon_z,t, \quad \epsilon_z,t \sim N(0, \sigma_z^2).$$

Intermediate firms face a Calvo-type price-setting friction: in each period $t$, a firm can re-optimize its intermediate-goods price with probability $(1 - \xi_p)$. Firms that cannot re-optimize index their price according to the steady-state inflation rate, $\pi$.

2.4 Employment

In each period $t$, a perfectly competitive representative employment agency hires labor from households to produce an aggregate labor service, $H_t$, using technology

$$H_t = \left[ \int_0^1 H_{i,t}^{\eta_w - 1} di \right]^{\eta_w - 1}.$$
where $H_{i,t}$ denotes the time $t$ input of labor service from household $i$ and $\theta_w$ controls the price elasticity of demand for each household’s labor service. The agency sells the aggregated labor input to the intermediate firms for a nominal price of $W_t$ per unit. Households (both capital holders and non-capital holders) face a Calvo-type wage-setting friction: In each period $t$, a household can re-optimize its nominal wage with probability $(1 - \xi_w)$. Households that cannot re-optimize index their wage according to the steady-state wage growth rate, $\gamma \pi$.

### 2.5 Aggregation, government, and resource constraint

Aggregate consumption, hours, investment, and capital are defined as:

- $C_t \equiv \chi C^n_t + (1 - \chi) C^c_t,$
- $H_t \equiv \chi H^n_t + (1 - \chi) H^c_t,$
- $I_t \equiv (1 - \chi) I^c_t,$
- $K_t \equiv (1 - \chi) K^c_t.$

The resource constraint is

$$C_t + I_t + G_t + \frac{\nu}{2} \left( \frac{B^n_t}{Y_t} \right)^2 = Y_t,$$

where $G_t$ is government spending.

The central bank follows a Taylor rule with interest-rate smoothing:

$$\frac{R_t}{\bar{R}} = \left( \frac{R_{t-1}}{\bar{R}} \right)^{\rho_R} \left\{ \left( \frac{\pi_t}{\pi} \right)^{\phi_\pi} \left( \frac{Y_t}{\gamma t Y} \right)^{\phi_Y} \right\}^{1 - \rho_R} \exp(\epsilon_{R,t}), \quad (11)$$

where $\rho_R$ is the persistence of the rule and and $\phi_\pi$ and $\phi_Y$ are the size of the policy response to the deviation of inflation and output from their steady states, respectively. $\epsilon_{R,t}$ is a monetary policy shock that follows $\epsilon_{R,t} \sim N(0, \sigma_R^2)$.

The government budget constraint is

$$T_t + G_t = B^R_t - R_{t-1} \frac{B^n_t}{\bar{R}} + \tau_{c,t} C_t + \tau_{h,t} \int_0^1 W_{i,t} H_{i,t} di + \tau_{k,t} (R^k_t - \delta) K_{t-1},$$

where $T_t \equiv \chi T^n_t + (1 - \chi) T^c_t$ and $B^R_t$ is the real government bond. The transfers are equally distributed across households: $T_t = T^n_t = T^c_t$. I assume that the real government bond is related to the previous period transfer according to

$$\hat{B}^R_t = \rho_B \hat{B}^R_{t-1} + \phi_{B,Y} \hat{Y}_{t-1} + \phi_{B,T} \hat{T}_{t-1},$$

where $\phi_{B,T}$ is restricted to a value that makes the government bond non-explosive. The lump-sum
transfers adjust so that the government budget constraint is satisfied. Finally, the bond market clearing condition is

\[ \chi B^n_t + (1 - \chi) B^c_t = B^g_t. \]

### 3 Solution and parameterization

#### 3.1 Solution

The equilibrium conditions that characterize the economy are listed in Appendix 5.1. To solve the model, I follow the methodology developed in Ilut et al. (2016). They analyze a class of dynamic models with ambiguity averse agents where agents differ in their worst-case beliefs and recognize that other agents have different worst cases. Because there is heterogeneity in the worst-case beliefs in the steady state, one needs to jointly solve for the steady state and the equilibrium decision rules. The brief outline of the solution method is as follows. First, I log-linearize the equilibrium conditions of the model. Second, I guess the elasticities that map from state variables to endogenous variables. Third, I jointly solve for the steady state and the dynamics taking into account the heterogeneity in worst-case beliefs. I iterate the second and the third steps until the guessed elasticities and the solution coincide. Finally, I verify that the guessed beliefs are indeed the worst-case beliefs by plugging in the decision rules into the linearized value functions. In my model, this can be done simply by checking that both capital holders and non-capital holders become worse off in terms of expected continuation utility when fiscal uncertainty increases. Appendix 5.2 provides more details.

#### 3.2 Bayesian impulse response matching estimation

The first step of the estimation is to conduct a structural VAR (SVAR) analysis similar to those in Fernández-Villaverde et al. (2015), where I augment the standard quarterly VAR with fiscal volatility processes. As in Fernández-Villaverde et al. (2015) and Born and Pfeifer (2014), I estimate the fiscal volatility processes (1) and (2) using the particle filter, where I use the tax rates and spending series of the consolidated government sector from Fernández-Villaverde et al. (2015). They construct average tax rates data from the national income and product accounts. Table 2 in Appendix 5.3 report the parameter estimates of the fiscal volatility processes.

I estimate four-lags recursive VARs with linear time trends where I include the following variables: (1) the filtered fiscal volatility, (2) log real per capita GDP, (3) log real per capita consumption, (4) log real per capita investment, (5) log per capita hours worked, (6) log real wage, (7) GDP deflator inflation, (8) Federal funds rate, and (9) log government-spending-to-GDP ratio. The ordering of

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Fernández-Villaverde et al. (2015) provide two evidences that indicate that the measured fiscal volatility series resemble actual changes in fiscal uncertainty faced by agents in the U.S. economy. First, spikes in fiscal volatility match historical events that are associated with heightened uncertainty about fiscal policy. Second, the fiscal volatility series is positively correlated with policy uncertainty index by Baker et al. (2016), which is constructed using a very different methodology.
the variables reflects the idea that fiscal uncertainty shocks are exogenous. I use filtered volatility, which conditions on information up to period \( t \), instead of smoothed volatility, which uses all the information available up to the end of sample, so that the timing is consistent with that of the VAR. I estimate four separate VARs for each fiscal instruments, where the only difference among the VARs is the fiscal volatility process that enters as observables.\(^6\) The sample period is 1980:Q1 – 2008:Q3. The sample starts after the Volcker appointment to the Federal Reserve chair in order to circumvent parameter instabilities related to monetary policy. Similarly, in order to avoid complications arising from the zero lower bound, I trim the observation after 2008:Q4.

I fix a small number of parameters prior to the estimation. The growth rate of labor-augmenting technological progress is set to \( \gamma = 1.004 \). For the capital share \( \alpha \), discount factor \( \beta \), and depreciation rate \( \delta \), I choose 0.3, 0.99, and 0.015, respectively. I set the bond holding cost \( v \) to a small value of 0.01; this helps close the model without substantially affecting its dynamic properties (Fernández-Villaverde et al. 2011). Regarding the parameters for the fiscal instrument processes (1) and (2), I use the estimates presented in Table 2 in Appendix 5.3. Regarding the government bond process, the steady-state debt-to-GDP ratio \( \bar{B}^g / \bar{Y} \) is set to 0.64, which is in line with the average debt-to-GDP ratio during the sample period. Parameters that controls the debt process \( \rho_B, \phi_{B,Y}, \) and \( \phi_{B,T} \) are set to 0.985, 0.002, and 0.000, respectively. These are based on the estimates from a regression of log government debt on log output and log transfers. The AR(1) parameter for the discount factor shock \( \rho_d \) is set to 0.5.

I estimate the remaining set of parameters using a Bayesian impulse-response-matching method, developed by Christiano et al. (2010a) and Christiano et al. (2010b). In what follows, I briefly outline the method. The first step is to compute the “likelihood” of the data from approximation based on conventional asymptotic distribution theory. Let \( \hat{\psi} \) denote the impulse response function computed from the VAR and let \( \psi(\theta) \) denote the impulse response function from the theoretical model, which depend on the structural parameters \( \theta \). \( \hat{\psi} \) and \( \psi(\theta) \) contain the VAR impulse responses of GDP, consumption, investment, hours worked, real wage, inflation, and the Federal fund rate and their theoretical counterparts. Suppose the theoretical model as well as the VAR are correctly specified. Denote \( \theta_0 \) and \( \psi(\theta_0) \) the true parameter vector and impulse response function, respectively. Then we have

\[
\sqrt{T}(\hat{\psi} - \psi(\theta_0)) \xrightarrow{d} N(0, Z(\theta_0)),
\]

where \( T \) is the length of the sample and \( Z(\theta_0) \) is the asymptotic sampling variance, which is a function of \( \theta_0 \). The asymptotic distribution of \( \hat{\psi} \) is rewritten as

\[
\hat{\psi} \xrightarrow{d} N(\psi(\theta_0), V), \quad V \equiv \frac{Z(\theta_0)}{T}.
\]

I use a consistent estimator of \( V \), where the main diagonal elements consist of the sample variance of

\(^6\)I also estimated a VAR where the fiscal volatility processes for all instruments enter jointly. The results are very similar.
ψ. Because of small sample considerations, I set the non-diagonal terms of V to zero. I then calculate the likelihood

\[ L(\psi | \theta) = (2\pi)^{-\frac{N}{2}} |V|^{-\frac{1}{2}} \exp\{-0.5[\hat{\psi} - \psi(\theta)]'V^{-1}[\hat{\psi} - \psi(\theta)]\}, \]

where \( N \) is the total number of elements in the impulse responses to be matched.\(^7\) Intuitively, the likelihood is higher when the theoretical impulse response \( \psi(\theta) \) is closer to the empirical counterpart \( \hat{\psi} \), taking into account the precision of the estimated empirical responses. From the Bayes law, the posterior distribution \( P(\theta | \psi) \) is

\[ P(\theta | \psi) = \frac{P(\theta)L(\psi | \theta)}{P(\psi)}, \]

where \( P(\theta) \) is the prior and \( P(\psi) \) is the marginal likelihood. I compute the posterior distribution using the random-walk Metropolis-Hastings algorithm.

The priors are reported in Table 1. Because most parameters are standard, I focus on two key parameters. First, I use several sources to determine the prior for the capital market participation rate. According to the household survey conducted in Investment Company Institute (2008), around 50 percent of U.S. households held stocks. In contrast, according to the Consumer Expenditure Survey (CEX) conducted by the Bureau of Labor Statistics, around 20 percent of households hold stocks (Vissing-Jørgensen 2002). The figure may or may not include indirect holdings. The Survey of Consumer Finances (SCF) by the Federal Reserve Board finds that 13.8 percent of U.S. households held stocks in 2013 but that figure rises to around 50 percent if we include indirect holdings (Bricker et al. 2014). While in reality it is not clear whether or not households who own stocks only indirectly and those who directly hold stocks behave similarly, I center my prior for the share of non-capital holders \( \chi \) around 0.5 (which implies the capital market participation rate of 50 percent) with a standard deviation of 0.1. Second, the parameter \( \eta \), which controls the size of the entropy constraint, determines the extent to which changes in volatility translate into changes in ambiguity. Ilut and Schneider (2014) suggest an upper-bound of the value of \( \eta \) to be 2, reflecting the idea that the level of ambiguity should not be “too large” in a statistical sense compared to the variability of the data. I re-parametrize and estimate \( 0.5\eta \), for which I set a Beta prior.

4 Quantitative analysis

I now present the main results. To facilitate comparison with the previous literature (Fernández-Villaverde et al. 2015 and Born and Pfeifer 2014), I begin by studying an uncertainty shock to the capital income tax rate.
Table 1: Estimated parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Prior</th>
<th>Posterior mode</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Baseline</td>
<td>Rep. agent</td>
</tr>
<tr>
<td>( \phi )</td>
<td>Inverse Frisch elasticity</td>
<td>( G(1, 0.5) )</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.03)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>( b )</td>
<td>Consumption habit</td>
<td>( B(0.3, 0.1) )</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.05)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>Investment adj. cost</td>
<td>( G(3, 0.5) )</td>
<td>4.29</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.21)</td>
<td>(0.57)</td>
</tr>
<tr>
<td>( \chi )</td>
<td>Non-capital holders share</td>
<td>( B(0.5, 0.1) )</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.02)</td>
<td></td>
</tr>
<tr>
<td>( \frac{1}{1-\xi_p} )</td>
<td>Avg. frequency of price adjustment</td>
<td>( G(3, 0.3) + 1 )</td>
<td>7.15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.15)</td>
<td>(0.41)</td>
</tr>
<tr>
<td>( \frac{1}{1-\xi_w} )</td>
<td>Avg. frequency of wage adjustment</td>
<td>( G(3, 0.3) + 1 )</td>
<td>4.21</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.24)</td>
<td>(0.44)</td>
</tr>
<tr>
<td>( \rho_R )</td>
<td>Interest smoothing</td>
<td>( B(0.65, 0.05) )</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.01)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>( \phi_\pi )</td>
<td>Inflation response</td>
<td>( N(1.5, 0.1) )</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.01)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>( \phi_Y )</td>
<td>Output response</td>
<td>( G(0.1, 0.05) )</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.001)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>0.5( \eta )</td>
<td>Entropy constraint</td>
<td>( B(0.5, 0.15) )</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.02)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>Log marginal likelihood</td>
<td></td>
<td>-630</td>
<td>-666</td>
</tr>
</tbody>
</table>

Notes: \( B \) refers to the Beta distribution, \( N \) to the Normal distribution, and \( G \) to the Gamma distribution. For the priors, the numbers inside the parentheses are the prior means and standard deviations. For the posteriors, the numbers inside the parentheses are the posterior standard deviations.

4.1 The capital income tax uncertainty shock

4.1.1 Amplification and co-movement

Table 1 reports the posterior estimates. For the baseline model with limited capital market participation, the estimates are in line with the New Keynesian literature, featuring relatively large real and nominal rigidities. For example, the estimated average frequencies of price and wage adjustments are 7 and 4 quarters, respectively. Later I examine the performance of the model when prices and wages are assumed to be flexible. The estimated share of capital holders is \( (1 - \chi =) 16\% \). The estimate is in line with the CEX data and the direct holdings rate in the SCF data. Below I check the robustness of the results by considering higher capital market participation rate. Figure 1 plots

---

\(^7\)In my context, since I match the responses of seven variables to uncertainty shocks for four different fiscal instruments, \( N = 7 \times 4 = 28 \).
Figure 1: Impulse responses to a capital income tax uncertainty shock: aggregate variables

Notes: The figure reports the impulse responses to a two-standard-deviation increase in $\sigma_{\tau_k}$ (capital income tax volatility). The units are in percents (annual percentage points for inflation and nominal rate). The black lines are the mean responses from the VAR and the shaded areas are the 95% confidence band. The blue lines with circles are the responses from the baseline heterogeneous agent model with limited capital market participation. The purple lines are the responses from the representative agent model with full capital market participation where we set the share of non-capital holders $\chi$ to 0, while holding other parameters at the estimated values from the limited participation model.

The VAR responses of aggregate variables to a two-standard-deviation increase in uncertainty about capital income tax as well as the estimated impulse response from the model. According to the VAR, an increase in capital income tax uncertainty generates a sizable contraction, causing an estimated 0.4 percentage mean decline in output a year after the shock and similar reductions in consumption, investment, and hours. The real wage and the nominal interest rate also fall and inflation declines mildly. The heterogeneous agent model with limited participation matches the VAR response quite well. As in the data, the model predicts substantial drops in output, consumption, investment, and hours. The model also replicates drops in prices such as real wages, inflation, and the nominal rate. To understand the mechanism of the heterogeneous agent model, in Figure 2, I report the paths of
Figure 2: Impulse responses to a capital income tax uncertainty shock: path of fiscal instruments under the true and worst-case DGPs

Notes: The figure reports the impulse responses to a two-standard-deviation increase in $\sigma_{\tau_k}$ (capital income tax volatility). The units are in percents. The blue lines with circles and the black lines are the responses under the true and the worst-case DGPs, respectively, from the heterogeneous agent model with limited capital market participation.

transfers, consumption, labor and capital income taxes in response to the capital income tax uncertainty shock (blue lines with circles). I also report impulse responses under the worst-case beliefs (black solid lines): they describe the worst-case expected path that agents worry about when uncertainty increases. The worst-case capital income tax rate is heterogeneous: capital holders’ worst case is high capital income tax while non-capital holders’ worst case is low capital income tax. Given the estimated fiscal rules, the capital holders’ high capital tax under the worst case translates into higher transfers due to increased government revenue but moves consumption and labor taxes little. For non-capital holders their worst-case scenario of low capital tax translates into low transfers.

Figure 3 reports the capital holders’ and non-capital holders’ consumption, hours, bond holdings, and total tax paid, which includes lump-sum transfers.\(^8\) For both capital and non-capital holders,

\[^8\]To be precise, total tax paid for the capital holders is given by $\tau_{c,t}C^c_t + \tau_{h,t} \int W_{i,t}H^c_i,t d\bar{t} + \tau_{k,t}(R^k_t - \delta)K^c_{t-1} - T^c_t$ and that of the non-capital holders is given by $\tau_{c,t}C^n_t + \tau_{h,t} \int W_{i,t}H^n_i,t d\bar{t} - T^n_t$.\]
Figure 3: Impulse responses to a capital income tax uncertainty shock: individual variables under the true and worst-case DGPs

Notes: See notes from Figure 2.

total tax paid increases under the worst-case belief. Due to the perceived negative income effect, non-capital holders immediately cut their consumption by sizable amount since they expect substantially lower consumption in the future. In the aggregate, the decline of consumption by non-capital holders outweighs the increase by capital holders and thus reduces aggregate demand and, through nominal rigidities, raises markups. This, in turn, depresses labor demand and lowers real wages. As a result, equilibrium employment and hence output drop substantially.

To highlight the role of limited capital market participation, I show in Figure 1 the impulse responses where I set the estimated share of non-capital holders (χ) to zero while holding other parameters at the original estimated values. In response to an increase in uncertainty about capital income tax, the representative household acts as if future tax will be high. Facing lower after-tax return on capital, they substitute away from investment and increase consumption. Low investment

\footnote{Capital holders receive back some portion of the higher capital income tax that they pay under the worst case scenario as increased transfers. However, they do not get all of them back because some of the capital tax paid will be redistributed to non-capital holders as transfers.}
Figure 4: Impulse responses to a capital income tax uncertainty shock: full capital market participation

Notes: The figure reports the impulse responses to a two-standard-deviation increase in $\sigma_{\tau_k}$ (capital income tax volatility). The units are in percents (annual percentage points for inflation and nominal rate). The black lines are the mean responses from the VAR and the shaded areas are the 95% confidence band. The blue lines with circles are the responses from the baseline heterogeneous agent model with limited capital market participation. The purple lines are the responses from the representative agent model with full capital market participation, where we re-estimate the parameters while imposing the constraint that the share of non-capital holders $\chi$ is 0.

reduces household’s incentive to supply labor. The increase in consumption, however, mitigates the decline in overall aggregate demand and thus equilibrium employment and output decline only mildly. At the same time, since hours fall without a large change in aggregate demand, markups also change little and hence real wages do not move. A different way to evaluate the contribution of limited capital market participation is to re-estimate the model, subject to the constraint $\chi = 0$. The resulting impulse responses are shown in Figure 4. To ease comparison, I also show the impulse responses from the estimated baseline limited capital market participation model. The representative agent model still has difficulty matching the VAR. For example, output and hours do not fall enough, consumption increases rather than decreases, and real wages do not decline. As a result, the limited capital market
Figure 5: Counterfactual impulse responses with a homogeneous worst-case scenario: aggregate variables

Notes: The figure reports the impulse responses to a two-standard-deviation increase in $\sigma_{\tau_k}$ (capital income tax volatility). The units are in percents (annual percentage points for inflation and nominal rate). The black lines are the mean responses from the VAR and the shaded areas are the 95% confidence band. The blue lines with circles are the responses from the baseline heterogeneous agent model with limited capital market participation. The purple lines are the responses from the representative agent model with full capital market participation where we set the share of non-capital holders $\chi$ to 0, while holding other parameters at the estimated values from the limited participation model. The dark green dashed lines are the impulse responses from the heterogeneous agent model where I counterfactually set the non-capital holders’ worst-case to the capital holders’.

participation model beats the representative agent model in terms of marginal likelihood by 36 log points (Table 1). These observations indicate that household heterogeneity due to limited capital market participation is a critical feature to explain the empirical effects of fiscal uncertainty shocks.

One potential concern about my finding is that household heterogeneity per se, rather than heterogeneous worst-case scenarios arising from household heterogeneity is driving the results. To address this concern, in Figure 5 I plot the impulse responses to a capital income tax uncertainty shock where I counterfactually set the worst-case scenarios for both capital and non-capital holders to a high capital tax. With a homogeneous worst-case scenario, the responses of output, consumption,
Figure 6: Counterfactual impulse responses with a homogeneous worst-case scenario: individual variables under the true and worst-case DGPs

Notes: The figure reports the impulse responses to a two-standard-deviation increase in $\sigma_{\tau_k}$ (capital income tax volatility). The units are in percents. The blue lines with circles and the black lines are the responses under the true and the worst-case DGPs, respectively, from the heterogeneous agent model with limited capital market participation where I counterfactually set the non-capital holders’ worst-case to the capital holders’.

hours are amplified in the medium run compared to the representative agent model. However, in the short run the homogeneous worst case model generates little amplification and consumption slightly increases. This is because, as shown in Figure 6 where I plot impulse response under the worst-case scenario, non-capital holders perceive a positive income effect due to the anticipation of lower total taxes under the worst-case DGP. I conclude that the worst-case heterogeneity is an important ingredient to generate a sizable contractionary effect to a capital income tax uncertainty shock.

4.1.2 Impulse responses estimated from the VAR on simulated data

Following Christiano et al. (2010a) and Christiano et al. (2010b), the target I use in the Bayesian impulse-response-matching method is the theoretical impulse response derived from the model. While this facilitates the mapping of the impulse responses from various counterfactuals to the fit of the data,
Figure 7: VAR impulse responses estimated using the simulated data generated from the model

**Notes:** The figure reports the impulse responses to a two-standard-deviation increase in $\sigma_{\tau_k}$ (capital income tax volatility). The units are in percents (annual percentage points for inflation and nominal rate). The blue lines with circles are the VAR responses estimated using artificial time series generated from the baseline heterogeneous agent model with limited capital market participation. The purple lines are the VAR responses estimated using artificial time series generated from the representative agent model with full capital market participation where we set the share of non-capital holders $\chi$ to 0, while holding other parameters at the estimated values from the limited participation model.

It is important to check whether the finite-order VAR representation provides a good approximation of the impulse responses of the theoretical model. To this end, I conduct a Monte-Carlo analysis by estimating 1,000 structural VARs, each using 115 quarters of artificial time series generated from the baseline limited capital market participation model. For the VAR, I use the exact same specification described in Section 3.2.\(^{10}\) In Figure 7, I plot the mean VAR impulse responses from the artificial time series generated from the baseline limited capital market participation model. For the technology shock, I set $\rho_z = 0.95$ and $100\sigma_z = 0.5$. For the discount factor shock, I set $\rho_d = 0.5$ and $100\sigma_d = 0.15$. Finally, the standard deviation of the monetary policy shock is set to $100\sigma_R = 0.03$. In the data, the standard deviations of output, consumption, and inflation are 1.28, 0.81, and 0.19, respectively. The model generates...
artificial time series averaged over 1,000 replications. For comparison, I also plot the VAR impulse responses estimated from artificial time series generated using the counterfactual representative agent economy, where I set the share of non-capital holders $\chi$ to 0 while holding other parameters at the estimated values in the limited capital market participation model. The VAR recovers the theoretical impulse responses quite well. In particular, the estimated VAR responses generate amplification and co-movement due to limited capital market participation. The VAR responses also replicate the reductions in real wages, inflation, and the nominal interest rate. Finally, the quantitative magnitudes of these VAR responses are in line with the theoretical impulse responses. These findings indicate that my impulse-response-matching procedure is successful in identifying and quantifying the propagation mechanism of fiscal uncertainty shocks.

4.1.3 Additional analysis

In this section, I analyze further the impact of capital income tax uncertainty shocks in the heterogeneous agent model by perturbing a model specification or parameter values.

One-sided uncertainty shocks.

So far the capital income tax uncertainty shocks I considered are two-sided: the upper and lower bounds of the belief set widens symmetrically. However, historical events suggest that changes in fiscal uncertainty are often one-sided in nature. The advantage of my model, which models ambiguity about future fiscal policies through intervals of means, compared to the standard model with only risk, which models fiscal uncertainty through mean-preserving spread in volatility, is that I can easily study the impact of one-sided uncertainty shocks. In Figure 8, I consider the impact of an increase in the upper bound of the belief set (an increase of up-side uncertainty). The size of the increase in upper bound is equal to that in the baseline specification of the two-sided uncertainty shock. Thus, the only difference from the two-sided case is that there is no change in the lower bound.11 First, in the representative agent model, since the worst case is high capital tax, the impulse response is identical to the case of a two-sided uncertainty shock. Second, even though the non-capital holders’ worst case is fixed in the heterogeneous agent model, the contraction is larger and generates co-movement. This is because non-capital holders amplify the New Keynesian demand effect caused by the capital holders’ reaction to the uncertainty shock. In Figure 9, I consider the impact of a lowering of the lower bound (an increase of down-side uncertainty). In the representative agent model, there is no effect since the worst-scenario for the representative agent is unchanged. In contrast, output, consumption and hours fall in the heterogeneous agent model. Finally, the output effect of the increase in up-side uncertainty is more persistent and is as five times as large as the increase in down-side uncertainty.

The role of capital market participation rate.

The standard deviations of output, consumption, and inflation of 1.09, 0.81, and 0.28, respectively.

11Note that this exercise is different from that of combining a two-sided uncertainty shock and a shift in the expectation. In the latter case, the worst-case scenarios for both agents change.

22
Notes: The figure reports the responses to a capital income tax uncertainty shock, but with the lower bound of the belief set fixed at the steady-state level. The units are in percents (annual percentage points for inflation and nominal rate). The blue lines with circles are the responses from the baseline heterogeneous agent model with limited capital market participation. The purple lines are the responses from the representative agent model with full capital market participation where we set the share of non-capital holders $\chi$ to 0, while holding other parameters at the estimated values from the limited participation model.

To examine how the rate of capital market participation affects the results, I re-estimate the limited capital market participation model while fixing the capital market participation rate to $1 - \chi = 0.5$. This value is near the upper bound of the evidence regarding the share of stock holders. In Figure 10, I plot the resulting impulse responses from the re-estimated heterogeneous agent model as well as its representative counterpart, where I set the share of non-capital holders to $\chi = 0$ while holding fixed other parameters. While the limited participation model cannot replicate the drop in the real wages, the model generates a simultaneous decline in output, hours, consumption, and investment as well as amplification relative to the representative agent model. Thus, the key properties of the heterogeneous agent model regarding macro quantities are robust to higher capital market participation rate.
Figure 9: One-sided uncertainty shock: an increase of down-side uncertainty

Notes: The figure reports the responses to a capital income tax uncertainty shock, but with the upper bound of the belief set fixed at the steady-state level. The units are in percents (annual percentage points for inflation and nominal rate). The blue lines with circles are the responses from the baseline heterogeneous agent model with limited capital market participation. The purple lines are the responses from the representative agent model with full capital market participation where we set the share of non-capital holders $\chi$ to 0, while holding other parameters at the estimated values from the limited participation model.

The role of price and wage rigidities.

To illustrate the role of sticky prices and wages, in Figure 11, I plot the impulse responses of the representative agent economy and the heterogeneous agent economy with flexible prices and wages, where I set the average frequencies of price and wage adjustments to $1/(1-\xi_p) = 1.01$ and $1/(1-\xi_w) = 1.01$. In the representative agent model, the flexible prices and wages specification (black dashed lines) produces slightly smaller impulse response compared to the baseline specification (purple lines). In the heterogeneous agent model, the impulse response in the flexible prices and wages case (dark green lines with crosses) is very similar to the representative agent version. This exercise underscores the importance of nominal rigidities: they magnify the negative income effects perceived by non-capital holders through countercyclical markups. This finding echoes conclusions from Fernández-Villaverde...
Notes: The figure reports the impulse responses to a two-standard-deviation increase in $\sigma_{\tau_k}$ (capital income tax volatility). The units are in percents (annual percentage points for inflation and nominal rate). The black lines are the mean responses from the VAR and the shaded areas are the 95% confidence band. The blue lines with circles are the responses from the heterogeneous agent model with limited capital market participation where the capital market participation rate is set to $1 - \chi = 0.5$. The purple lines are the responses from the representative agent model with full capital market participation where we set the share of non-capital holders $\chi$ to 0, while holding other parameters at the estimated values from the limited participation model.

et al. (2015) and Basu and Bundick (2017) which emphasize countercyclical markups due to nominal rigidities in the transmission of uncertainty shocks.

Effect of the zero lower bound.

Finally, I analyze the impact of a capital income tax uncertainty shock when the economy is stuck at the zero lower bound (ZLB) on the nominal interest rate. This is a highly relevant exercise since, as noted by Baker et al. (2016) and Fernández-Villaverde et al. (2015) and others, the post-Great Recession period in the U.S. has experienced high fiscal uncertainty combined with the ZLB. To solve the economy at the zero lower bound, I extend the methodology used by Cagliairini and Kulish (2013)
Notes: The figure reports the impulse responses to a two-standard-deviation increase in $\sigma_{\tau_k}$ (capital income tax volatility). The units are in percents (annual percentage points for inflation and nominal rate). The black lines are the mean responses from the VAR and the shaded areas are the 95% confidence band. The blue lines with circles are the responses from the heterogeneous agent model with limited capital market participation. The dark green lines with crosses are the flexible prices and wages version of this economy. The purple lines are the responses from the representative agent model with full capital market participation where we set the share of non-capital holders $\chi$ to 0, while holding other parameters at the estimated values from the limited participation model. The black dashed lines are the flexible prices and wages version of this economy.

and Del Negro et al. (2015) to accommodate ambiguity aversion and heterogeneous worst cases. In Appendix 5.2 I describe the solution procedure in detail.

To compute the impulse responses to capital income tax uncertainty shocks under the ZLB, I follow the procedure used in Fernández-Villaverde et al. (2015). First, I hit the economy with an innovation to the household discount factor at period $t_1$ so that the economy is at the ZLB for $t_1 \leq t \leq t_2$. I choose the size of the innovation so that the economy is at the ZLB for five quarters. I then compare the path of endogenous variables in this economy with another economy where it experiences at period $t_1$ not only the discount factor shock that forces the economy to the ZLB but
Notes: The figure reports the impulse responses to a two-standard-deviation increase in $\sigma_{\tau_k}$ (capital income tax volatility). The units are in percents (annual percentage points for inflation and nominal rate). The nominal interest rate is expressed in terms of levels rather than the deviation from the steady state. The blue lines with circles are the responses from the baseline heterogeneous agent model with limited capital market participation. The purple lines are the responses from the representative agent model with full capital market participation where we set the share of non-capital holders $\chi$ to 0, while holding other parameters at the estimated values from the limited participation model. Both the blue lines with circles and the purple lines are the responses under the Taylor rule. The red lines with crosses and the black dashed lines are the respective responses under the zero lower bound.

also a two-standard deviation increase in capital income tax uncertainty.\textsuperscript{12} The difference between the path of endogenous variables between the two economies thus allows me to isolate the effect of a fiscal uncertainty shock when the economy is already at the ZLB. In Figure 12, I plot such impulse responses beginning from $t = t_1$ except for the nominal interest rate, for which I report the actual realized path under the ZLB.

Comparing the baseline heterogeneous agent model under the Taylor rule and the baseline model under the ZLB, the output response of the latter is much larger than the response of the former. For

\textsuperscript{12}While the algorithm allows the capital income tax uncertainty shock to alter the length of the ZLB, it turns out that the duration of the ZLB is the same with or without the uncertainty shock.
example, under the ZLB the output drops as much as around 1.4 percent, while under the Taylor rule output drops by 0.3 percent. In my model, the effects of capital tax uncertainty are much larger because the central bank cannot lower the interest rate to counteract low aggregate demand due to the consumption cut by non-capital holders.

4.2 Uncertainty shocks for other fiscal instruments

Household heterogeneity is important for understanding the transmission of uncertainty shocks to other fiscal instruments. In Figure 13, I plot the VAR mean responses of aggregate variables to a two-standard-deviation increase in uncertainty about consumption tax as well as the estimated impulse response from the model. In the data, a consumption tax uncertainty shock generates sizable reductions in output, consumption, and hours worked. While the macro quantities barely move in the full market participation model, the limited capital market participation model is able to generate contractions in output, consumption, and hours that are closer to the data.\(^{13}\)

To understand this, in Figures 14 and 15, I report the paths of fiscal variables and individual variables, respectively, under the true DGP and the worst-case beliefs. As we saw in the case of the capital income tax uncertainty shock, the worst-case consumption tax is heterogeneous: capital holders fear high consumption tax while non-capital holders fear low tax. This worst-case heterogeneity is due to the fact that capital holders consume more than non-capital holders. For capital holders, the high consumption tax translates into higher total tax paid over the medium run under their worst-case scenario. For non-capital holders, even though their consumption tax is lower under the worst-case belief, this lower tax translates into lower transfers due to lower government revenue and thus their total tax paid increases under the worst-case scenario. Due to the perceived negative income effect, non-capital holders cut their consumption and thus aggregate demand falls. Due to nominal rigidities, this low demand translates into higher markups and thus lower equilibrium hours and output.\(^{14}\)

In contrast to capital income tax uncertainty and consumption tax uncertainty shocks, as I show in Figures 17 and 19 in the Appendix, household heterogeneity does not lead to large amplification for government spending and labor income tax uncertainty shocks. This is because for both shocks the worst-case scenarios are homogeneous. In the case of the government spending uncertainty shock, since government spending is mainly financed by a lump-sum tax that is imposed equally on both types of households, the worst-case scenario of both capital holders and non-capital holders is a higher government spending (Figure 18 in the Appendix). For the labor income tax uncertainty shock, the worst case is a higher tax for both agents since an increase in the labor income tax generates a recession.

\(^{13}\)In Figure 13, I plot the impulse responses where I set the share of non-capital holders \(\chi\) to zero while holding other parameters at the estimated values. I have also re-estimated the model subject to the constraint \(\chi = 0\). The impulse response is very similar and hence is not shown.

\(^{14}\)The success of the limited capital market participation model is driven entirely by the worst-case heterogeneity. Indeed, as shown in Figure 16 in the Appendix, when we counterfactually set the non-capital holders’ worst-case scenario to a high consumption tax, an increase in consumption tax uncertainty generates a boom instead of a recession.
Figure 13: Impulse responses to a consumption tax uncertainty shock: aggregate variables

Notes: The figure reports the impulse responses to a two-standard-deviation increase in $\sigma_{r_c}$ (consumption tax volatility). The units are in percents (annual percentage points for inflation and nominal rate). The black lines are the mean responses from the VAR and the shaded areas are the 95% confidence band. The blue lines with circles are the responses from the baseline heterogeneous agent model with limited capital market participation. The purple lines are the responses from the representative agent model with full capital market participation where we set the share of non-capital holders $\chi$ to 0, while holding other parameters at the estimated values from the limited participation model.
Figure 14: Impulse responses to a consumption tax uncertainty shock: path of fiscal instruments under the true and worst-case DGPs

Notes: The figure reports the impulse responses to a two-standard-deviation increase in $\sigma_{\tau_c}$ (consumption tax volatility). The units are in percents. The blue lines with circles and the black lines are the responses under the true and the worst-case DGPs, respectively, from the heterogeneous agent model with limited capital market participation.
Figure 15: Impulse responses to a consumption tax uncertainty shock: individual variables under the true and worst-case DGPs

Notes: See notes from Figure 14.
and this aggregate effect outweighs the redistribution generated by changes in taxes (Figure 20 in the Appendix). As a result, the two fiscal uncertainty shocks do not feature a strong negative income effect associated with capital tax and consumption tax uncertainty shocks. In turn, this implies that household heterogeneity do not substantially magnify the impact of government spending and labor income tax uncertainty shocks. The results in this subsection thus demonstrates that whether the limited capital market participation significantly alters the the transmission of a fiscal uncertainty shock or not depends critically on whether the fiscal instrument is redistributive or not.

5 Conclusion

What can explain the large impact of changes in uncertainty about fiscal policy on economic activity? In this paper, I proposed a transmission mechanism that magnifies the recessionary effect of an increase in fiscal uncertainty and demonstrate that it can account for some salient empirical features of the responses of macroeconomic variables to fiscal uncertainty shocks. In particular, I have shown that the limited capital market participation model captures agents’ concern about redistribution due to capital income and consumption tax changes that is absent in representative agent models. In my model, this redistribution concern has first-order effects because it shows up as heterogeneous worst-case beliefs that are parameterized by intervals of conditional means.
References


Appendix

5.1 Equilibrium conditions

In this section I report the equations that characterize the equilibrium of the estimated model presented in Section 2. First, the variables have to be scaled in order to induce stationarity. The variables are scaled as follows:

\[ c_t^c = \frac{C_t^c}{\gamma_t}, \quad i_t^c = \frac{I_t^c}{\gamma_t}, \quad b_t^c = \frac{B_t^c}{\gamma_t}, \quad k_t^c = \frac{K_{t-1}^c}{\gamma_t}, \quad t_t^c = \frac{T_t^c}{\gamma_t}, \quad \lambda_t^c = \gamma_t \Lambda_t^c, \quad q_t = \gamma_t Q_t, \]

\[ c_t^n = \frac{C_t^n}{\gamma_t}, \quad b_t^n = \frac{B_t^n}{\gamma_t}, \quad t_t^n = \frac{T_t^n}{\gamma_t}, \quad \lambda_t^n = \gamma_t \Lambda_t^n, \]

\[ w_t = \frac{W_t}{\gamma_t}, \quad k_{t-1} = \frac{K_{t-1}}{\gamma_t}, \quad c_t = \frac{C_t}{\gamma_t}, \quad i_t = \frac{I_t}{\gamma_t}, \quad t_t = \frac{T_t}{\gamma_t}, \quad \lambda_t = \frac{\Lambda_t}{\gamma_t}, \quad q_t = \frac{Q_t}{\gamma_t}, \quad b_t = \frac{B_t}{\gamma_t}, \]

where \( \Lambda_t^c, \quad Q_t, \quad \Lambda_t^n \) are the lagrangian multipliers for the budget constraint for the capital holders (7), the capital accumulation equation (8), and the budget constraint for non-capital holders (10), respectively. I use stars to denote conditional expectations under the worst-case beliefs.

Capital holders’ marginal utility:

\[ (1 + \tau_{c,t}) \lambda_t^c = \frac{1}{c_t^c - b \gamma^{-1} c_{t-1}^c} - \beta d_t E_t^c \left( \frac{b}{\gamma_{c_{t+1}} - b c_t^c} \right) \tag{12} \]

Bond decision (FONC w.r.t. \( B_t^c \)):

\[ \gamma \lambda_t^c = \beta d_t E_t^c \lambda_{t+1}^c \frac{R_t}{\pi_{t+1}} \tag{13} \]

Capital accumulation decision (FONC w.r.t. \( K_t^c \)):

\[ \gamma q_t = \beta d_t E_t^c \left[ \lambda_{t+1}^c \left\{ (1 - \tau_{k,t+1}) R_{t+1}^k + \tau_{k,t+1} \delta \right\} + q_{t+1} (1 - \delta) \right] \tag{14} \]

with the law of motion for capital:

\[ \gamma k_t^c = (1 - \delta) k_{t-1}^c + \left\{ 1 - \frac{\kappa}{2} \left( \frac{\gamma_i^c}{i_{t-1}} - \gamma \right)^2 \right\} i_t^c \tag{15} \]

Investment decision (FONC w.r.t. \( I_t^c \)):

\[ \gamma \lambda_t^c = \gamma q_t \left\{ 1 - \frac{\kappa}{2} \left( \frac{\gamma_i^c}{i_{t-1}} - \gamma \right)^2 \right\} - \kappa \left( \frac{\gamma_i^c}{i_{t-1}} - \gamma \right) \frac{\gamma_i^c}{i_{t-1}} + \beta d_t E_t^c q_{t+1} \kappa \left( \frac{\gamma_i^c}{i_{t-1}} - \gamma \right) \left( \frac{\gamma_i^c}{i_{t-1}} \right)^2 \tag{16} \]
Conditions associated with capital holders’ sticky wages:

\[ f_{c,t}^1 = f_{c,t}^2, \]  
\[ f_{c,t}^1 = (w_{c,t}^*)^{1-\theta_w} \lambda_i^c (1-\tau_{h,t}) H_t w_t + \xi_w \beta d_t E_t^c \left( \frac{\pi w_{c,t}^*}{\pi_{t+1} w_{c,t+1}^*} \right)^{1-\theta_w} f_{c,t+1}^1, \]  
\[ f_{c,t}^2 = \frac{\theta_w}{\theta_w - 1} (w_{c,t}^*)^{-\theta_w(1+\phi)} H_t^{1+\phi} + \xi_w \beta d_t E_t^c \left( \frac{\pi w_{c,t}^*}{\pi_{t+1} w_{c,t+1}^*} \right)^{-\theta_w(1+\phi)} f_{c,t+1}^2, \]  
\[ \pi_t^w = \pi_t w_t / w_{t-1} \]

Aggregate hours of capital holders is given by

\[ H_t^c = \int_0^1 \left( \frac{w_{i,c,t}}{w_t} \right)^{-\theta_w} H_t \, di \]
\[ = (w_{c,t}^{'})^{-\theta_w} H_t, \]  
where
\[ w_{c,t}^' = \left[ \int_0^1 (w_{i,c,t})^{-\theta_w} \, di \right]^{-\frac{1}{\theta_w}} \]

and additionally we have

\[ (w_{c,t}^{'})^{-\theta_w} = (1 - \xi_w) (w_{c,t}^*)^{-\theta_w} + \xi_w \left( \frac{\pi w_{c,t}^{'-1}}{\pi_t^w} \right)^{-\theta_w}. \]  

Total wages of capital holders is given by

\[ \int_0^1 w_{i,c,t} H_t^c \, di = \int_0^1 w_{i,c,t} \left( \frac{w_{i,c,t}}{w_t} \right)^{-\theta_w} H_t \, di \]
\[ = w_t H_t (w_{c,t}^*)^{1-\theta_w}, \]  
where
\[ w_{c,t}^* = \left[ \int_0^1 (w_{i,c,t})^{-\theta_w} \, di \right]^{1-\theta_w} \frac{1}{\pi_t^w}, \]

and additionally we have

\[ (w_{c,t}^*)^{1-\theta_w} = (1 - \xi_w) (w_{c,t}^*)^{1-\theta_w} + \xi_w \left( \frac{\pi w_{c,t}^{'-1}}{\pi_t^w} \right)^{1-\theta_w}. \]  

Conditions that relate input demands to factor prices:

\[ w_t = mc_t (1 - \alpha) \frac{y_t}{h_t}, \]
\[ R_t^k = mc_t \alpha \frac{y_t}{k_{t-1}}, \]
where \( mc_t \) is the real marginal cost.

Conditions associated with sticky prices:

\[
p^*_t = \left( \frac{\theta_p}{\theta_p - 1} \right) \frac{P^n_t}{P^d_t},
\]

\[
P^n_t = \lambda_t mc_t y_t + \xi_p \beta d_t E_t^c \left( \frac{\pi_{t+1}}{\pi_t} \right)^{\theta_p} P^n_{t+1},
\]

\[
P^d_t = \lambda_t c y_t + \xi_p \beta d_t E_t^c \left( \frac{\pi_{t+1}}{\pi_t} \right)^{1-\theta_p} P^d_{t+1},
\]

\[
1 = (1 - \xi_p) (p^*_t)^{1-\theta_p} + \xi_p \left( \frac{\pi}{\pi_t} \right)^{1-\theta_p},
\]

\[
\dd_t = (\dd_t)^{-\theta_p} y_t,
\]

\[
\dd_t = (1 - \xi_p) (p^*_t)^{-\theta_p} + \xi_p \left( \frac{\pi}{\pi_t} \right)^{-\theta_p},
\]

where the last two equations are the aggregation due to Calvo pricing.

Non-capital holders’ marginal utility:

\[
(1 + \tau_{ct}) \lambda^n_t = \frac{1}{c^n_t - b \gamma^{-1} c^n_{t-1}} - \beta d_t E^n_t \left( \frac{b}{\gamma^{c^n_t - b c^n_t}} \right)
\]

Bond decision (FONC w.r.t. \( B^n_t \)):

\[
\gamma \lambda^n_t \left( 1 + \frac{b^n_t}{y_t} \right) = \beta d_t E^n_t \lambda^n_{t+1} \frac{R_t}{\pi_{t+1}}
\]

Non-capital holders’ budget constraint:

\[
(1 + \tau_{ct}) c^n_t + b^n_t = (1 - \tau_{ht}) \int_0^1 w_{i, n, t} H^n_i d i + R_{t-1} b^n_{t-1} \frac{b^n_{t-1}}{\gamma^{c^n_{t-1}}} + t_t - \frac{y_t}{2} \left( \frac{b^n_t}{y_t} \right)^2
\]

Conditions associated with non-capital holders’ sticky wages:

\[
f^{1}_{n, t} = f^{2}_{n, t},
\]

\[
f^{1}_{n, t} = (w^{*}_{n, t})^{1-\theta_w} \lambda^n_t (1 - \tau_{ht}) H_t w_t + \xi_w \beta d_t E^n_t \left( \frac{\pi w^{*}_{n, t}}{\pi_{t+1} w^{*}_{n, t+1}} \right)^{1-\theta_w} f^{1}_{n, t+1},
\]

\[
f^{2}_{n, t} = \frac{\theta_w}{\theta_w - 1} (w^{*}_{n, t})^{-\theta_w (1+\phi)} H_t^{1+\phi} + \xi_w \beta d_t E^n_t \left( \frac{\pi w^{*}_{n, t}}{\pi_{t+1} w^{*}_{n, t+1}} \right)^{-\theta_w (1+\phi)} f^{2}_{n, t+1}
\]
Aggregate hours of capital holders is given by

\[ H^n_t = \int_0^1 \left( \frac{w_{i,n,t}}{w_t} \right)^{-\theta_w} H_t \, di \]

\[ = (w'_{n,t})^{-\theta_w} H_t, \tag{39} \]

where

\[ w'_{n,t} = \left[ \int_0^1 (w_{i,n,t})^{-\theta_w} \, di \right]^{-\frac{1}{\theta_w}}, \]

and additionally we have

\[ (w'_{n,t})^{-\theta_w} = (1 - \xi_w)(w^{*}_{n,t})^{-\theta_w} + \xi_w \left( \frac{\pi w'_{n,t-1}}{\pi^w_t} \right)^{-\theta_w}. \tag{40} \]

Total wages of capital holders is given by

\[ \int_0^1 w_{i,n,t} H^n_{i,t} \, di = \int_0^1 w_{i,n,t} \left( \frac{w_{i,n,t}}{w_t} \right)^{-\theta_w} H_t \, di \]

\[ = w_t H_t (w^{*}_{n,t})^{1-\theta_w}, \tag{41} \]

where

\[ w^{*}_{n,t} = \left[ \int_0^1 (w_{i,n,t})^{1-\theta_w} \, di \right]^{\frac{1}{1-\theta_w}}, \]

and additionally we have

\[ (w^{*}_{n,t})^{1-\theta_w} = (1 - \xi_w)(w^{*}_{n,t})^{1-\theta_w} + \xi_w \left( \frac{\pi w^{*}_{n,t-1}}{\pi^w_t} \right)^{1-\theta_w}. \tag{42} \]

Production function:

\[ \tilde{y}_t = k_{t-1}^{\alpha} h_t^{1-\alpha} \tag{43} \]

Monetary policy rule:

\[ \frac{R_t}{\bar{R}} = \left( \frac{R_{t-1}}{\bar{R}} \right)^{\rho_R} \left\{ \frac{\pi_t}{\bar{\pi}} \right\}^{\phi_n} \left( \frac{\bar{y}_t}{\bar{Y}} \right)^{\phi_Y} \right\}^{1-\rho_R} \tag{44} \]

Government budget constraint:

\[ t_t = b_{t-1}^g - \frac{R_{t-1}}{\gamma \pi_t} b_{t-1}^g + \tau_c t_c + \tau_h t_h + \chi \int_0^1 w_{i,n,t} H^n_{i,t} \, di + (1 - \chi) \int_0^1 w_{i,c,t} H^c_{i,t} \, di + \tau_{k,t}(R_t^k - \delta) k_{t-1} - g_t y_t \tag{45} \]

Government debt process:

\[ \hat{b}_t^g = \rho_B \hat{b}_{t-1}^g + \phi_{B,Y} \hat{y}_{t-1} + \phi_{B,T} \hat{t}_{t-1} \tag{46} \]
Aggregation:

\[ c_t = \chi c_{t-1} + (1 - \chi)c_t, \quad (47) \]
\[ H_t = \chi H_{t-1} + (1 - \chi)H_t, \quad (48) \]
\[ i_t = (1 - \chi)i_t, \quad (49) \]
\[ k_t = (1 - \chi)k_t. \quad (50) \]

Resource constraint:

\[ c_t + i_t + g_t y_t + \frac{v}{2} \left( \frac{b^n_t}{y_t} \right)^2 y_t = y_t \quad (51) \]

Bond-market clearing condition:

\[ \chi b^n_t + (1 - \chi) b^c_t = b^q_t \quad (52) \]

The 41 endogenous variables we solve are:

\[ y_t, c_t, c^n_t, c_t, \lambda^n_t, \lambda_t, q_t, k^c_t, k_t, i_t, H^c_t, H^n_t, H_t, f^1_c, f^2_c, w^*_c, \pi^w_t, w^t_c, w_t^t, b^1_t, b^2_t, \int_0^1 w_{i,c,t} H^c_{i,t} di, \int_0^1 w_{i,n,t} H^n_{i,t} di. \]

We have listed 41 equilibrium conditions above from (12) to (52).

5.2 Solution method

In this section, I first describe how to compute the solution of the model when the economy is not in the ZLB. In the second part, I describe how to derive the solution under the ZLB. For this I utilize the procedure by Cagliarini and Kulish (2013). Although the first part closely follows Ilut et al. (2016), I include it in the interest of completeness and because it is useful to understand the solution under the ZLB.

Let \( Y_t \) denote a \( n \times 1 \) vector of endogenous variables and \( Z_t \) a \( k \times 1 \) vector of exogenous state variables. The system of equilibrium conditions are composed of three types of equations. The first are the equations that describe the evolution of endogenous variables but do not involve expectations:

\[ f(Y_t, Y_{t-1}, Z_t) = 0 \]

Then there are equations that involves expectations. We explicitly distinguish between different agents’ belief set on which expectations are based on. Suppose there are \( m_i \) equations for each agent \( i = c, n \) (capital-holders and non-capital holders). Then there are total of \( \sum_i m_i = m \) equations:

\[ E^i_t [g^i(Y_t, Y_{t-1}, Y_{t+1}, Z_t)] = 0 \]
Finally, there are $k$ equations that characterize the law of motion of exogenous variables:

$$\ln Z_t = (I - P) \ln Z + P \ln Z_{t-1} + \epsilon_t$$

Agents' expected $Z_{t+1}$ under the worst-case belief is given by

$$E_t^i \ln Z_{t+1} = (I - P) \ln Z + P \ln Z_t + A^i \ln Z_t$$

where the matrix $A^i$ determines the belief adjustment relative to the true law of motion above. In my model, $Z_t$ contains the bound $a_{x,t}$ for each fiscal instruments and hence I use different matrices $A^i$ to pick different worst-case scenario for each agent.

To compute the solution of the model outside the ZLB, we follow the steps below:

1. Guess the elasticities, $\epsilon_{yy}$ and $\epsilon_{yz}$, of endogenous variables $Y$ with respect to endogenous and exogenous variables, respectively.

2. Compute the candidate steady state $\bar{Y}$ by evaluating the equations

$$f(\bar{Y}, \bar{Y}, \bar{Z}) = 0$$
$$g^i(\bar{Y}, \bar{Y}, \exp(\epsilon_{yz} A^i \ln \bar{Z}), \bar{Z}) = 0$$

3. Solve for the elasticities at the steady state by considering the log-linearized equilibrium conditions

$$G^0 \hat{Y}_t = G^1 \hat{Y}_{t-1} + G^2 E_t^i \hat{Y}_{t+1} + \Psi \hat{Z}_t$$

To solve for the worst-case expectations, we use

$$E_t^i \hat{Y}_{t+1} = \epsilon_{yy} \hat{Y}_t + \epsilon_{yz} (P + A^i) \hat{Z}_t$$

and hence we have

$$G^0 \hat{Y}_t = G^1 \hat{Y}_{t-1} + G^2 \epsilon_{yy} \hat{Y}_t + [G^2 \epsilon_{yz} (P + A^i) + \Psi] \hat{Z}_t$$

The solution is given by

$$\hat{Y}_t = \tilde{\epsilon}_{yy} \hat{Y}_{t-1} + \tilde{\epsilon}_{yz} \hat{Z}_t$$

where, using undetermined coefficients,

$$\tilde{\epsilon}_{yy} = (G^0 - G^2 \epsilon_{yy})^{-1} G^1$$
$$\tilde{\epsilon}_{yz} = (G^0 - G^2 \epsilon_{yy})^{-1} [G^2 \epsilon_{yz} (P + A^i) + \Psi]$$

4. Check that the resulting elasticities, $\tilde{\epsilon}_{yy}$ and $\tilde{\epsilon}_{yz}$, coincide with the initial guesses, $\epsilon_{yy}$ and $\epsilon_{yz}$. 

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If not, set the new guesses to $\varepsilon_{yy} = \tilde{\varepsilon}_{yy}$ and $\varepsilon_{yz} = \tilde{\varepsilon}_{yz}$ and return to step 1.

5. Verify that agents are indeed forming expectations under the worst-case beliefs. This can be done by checking that the expected continuation value for each agent, $E^c_i V_{t+1}^c$ and $E^n_i V_{t+1}^n$, decreases as uncertainty for each fiscal instrument increases.

I now consider the solution under the ZLB. Suppose in period $t$ agents expect that the interest rate is at the ZLB for $H$ periods ($t = 1, \ldots, H$) and then reverts back to the normal monetary policy rule (11) afterwards ($t = H + 1, \ldots$). The log-linearized equations during the ZLB are given by

$$G^0 \hat{Y}_t = G^1 \hat{Y}_{t-1} + G^2 E^i_{t} \hat{Y}_{t+1} + \Psi \hat{Z}_t,$$

(53)

where the log-linearized version of the policy rule (11) is replaced with $\hat{R}_t = -\bar{R}$. For periods $t = 1, \ldots, H$, the decision rule takes a time-varying form

$$\hat{Y}_t = \varepsilon_{yy,t} \hat{Y}_{t-1} + \varepsilon_{yz,t} \hat{Z}_t$$

which implies

$$E^i_{t} \hat{Y}_{t+1} = \varepsilon_{yy,t+1} \hat{Y}_t + \varepsilon_{yz,t+1} (P + A^i) \hat{Z}_t$$

(54)

From (53) and (54) we use method of undetermined coefficients to obtain

$$\varepsilon_{yy,t} = (G^0 - G^2 \varepsilon_{yy,t+1})^{-1} G^1$$

$$\varepsilon_{yz,t} = (G^0 - G^2 \varepsilon_{yy,t+1})^{-1} [G^2 \varepsilon_{yz,t+1} (P + A^i) + \Psi]$$

starting from $\varepsilon_{yy,H+1} = \varepsilon_{yy}$ and $\varepsilon_{yz,H+1} = \varepsilon_{yz}$. Lastly we check at each period during the ZLB ($t = 1, \ldots, H$) agents are forming expectations under the worst-case beliefs.
### 5.3 Additional figures and tables

#### Table 2: Estimated fiscal rules and shocks

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</table>

*Notes:* The Table reports posterior means of the fiscal volatility processes (1) and (2) from the particle filter. Following Fernández-Villaverde et al. (2015), I assume flat priors with loose boundary constraints. The numbers in parentheses are the 90% intervals. The sample period is 1970:Q2–2014:Q2.
Figure 16: Counterfactual impulse responses with a homogeneous worst-case scenario: consumption tax uncertainty shock

Notes: The figure reports the impulse responses to a two-standard-deviation increase in $\sigma_{\tau_c}$ (consumption tax volatility). The units are in percents (annual percentage points for inflation and nominal rate). The black lines are the mean responses from the VAR and the shaded areas are the 95% confidence band. The blue lines with circles are the responses from the baseline heterogeneous agent model with limited capital market participation. The purple lines are the responses from the representative agent model with full capital market participation where we set the share of non-capital holders $\chi$ to 0, while holding other parameters at the estimated values from the limited participation model. The dark green dashed lines are the impulse responses from the heterogeneous agent model where I counterfactually set the non-capital holders’ worst-case to the capital holders’.
Figure 17: Impulse responses to a government spending uncertainty shock

Notes: The figure reports the impulse responses to a two-standard-deviation increase in $\sigma_g$ (government spending volatility). The units are in percents (annual percentage points for inflation and nominal rate). The black lines are the mean responses from the VAR and the shaded areas are the 95% confidence band. The blue lines with circles are the responses from the baseline heterogeneous agent model with limited capital market participation. The purple lines are the responses from the representative agent model with full capital market participation where we set the share of non-capital holders $\chi$ to 0, while holding other parameters at the estimated values from the limited participation model.
Figure 18: Impulse responses to a government spending uncertainty shock: path of fiscal instruments under the true and worst-case DGPs

Notes: The figure reports the impulse responses to a two-standard-deviation increase in $\sigma_g$ (government spending volatility). The units are in percents. The blue lines with circles and the black lines are the responses under the true and the worst-case DGPs, respectively, from the heterogeneous agent model with limited capital market participation.
Figure 19: Impulse responses to a labor income tax uncertainty shock

**Notes**: The figure reports the impulse responses to a two-standard-deviation increase in $\sigma_{\tau_h}$ (labor income tax volatility). The units are in percents (annual percentage points for inflation and nominal rate). The black lines are the mean responses from the VAR and the shaded areas are the 95% confidence band. The blue lines with circles are the responses from the baseline heterogeneous agent model with limited capital market participation. The purple lines are the responses from the representative agent model with full capital market participation where we set the share of non-capital holders $\chi$ to 0, while holding other parameters at the estimated values from the limited participation model.
Figure 20: Impulse responses to a labor income tax uncertainty shock: path of fiscal instruments under the true and worst-case DGPs

Notes: The figure reports the impulse responses to a two-standard-deviation increase in $\sigma_{\tau}$ (labor income tax volatility). The units are in percents. The blue lines with circles and the black lines are the responses under the true and the worst-case DGPs, respectively, from the heterogeneous agent model with limited capital market participation.
5.4 Data sources

For macroeconomic variables, I use the following:

1. Real GDP in chained dollars, BEA, NIPA table 1.1.6, line 1.
2. GDP, BEA, NIPA table 1.1.5, line 1.
3. Personal consumption expenditures on nondurables, BEA, NIPA table 1.1.5, line 5.
4. Personal consumption expenditures on services, BEA, NIPA table 1.1.5, line 6.
5. Gross private domestic fixed investment (nonresidential and residential), BEA, NIPA table 1.1.5, line 8.
6. Personal consumption expenditures on durable goods, BEA, NIPA table 1.1.5, line 4.
7. Nonfarm business hours worked, BLS PRS85006033.
9. Civilian noninstitutional population (16 years and over), BLS LNU00000000.
10. Effective federal funds rate, Board of Governors of the Federal Reserve System.

I then conduct the following transformations of the above data:

11. Real per capita GDP: \((1)/(9)\)
12. GDP deflator: \((2)/(1)\)
13. Real per capita consumption: \([(3)+(4)]/[(9) \times (12)]\)
14. Real per capita investment: \([(5)+(6)]/[(9) \times (12)]\)
15. Per capita hours: \((7)/(9)\)
16. Real wages: \((8)/(12)\)